

# 

### Semester Two Examination, 2017

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNITS 3 AND 4**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student Number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 97 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

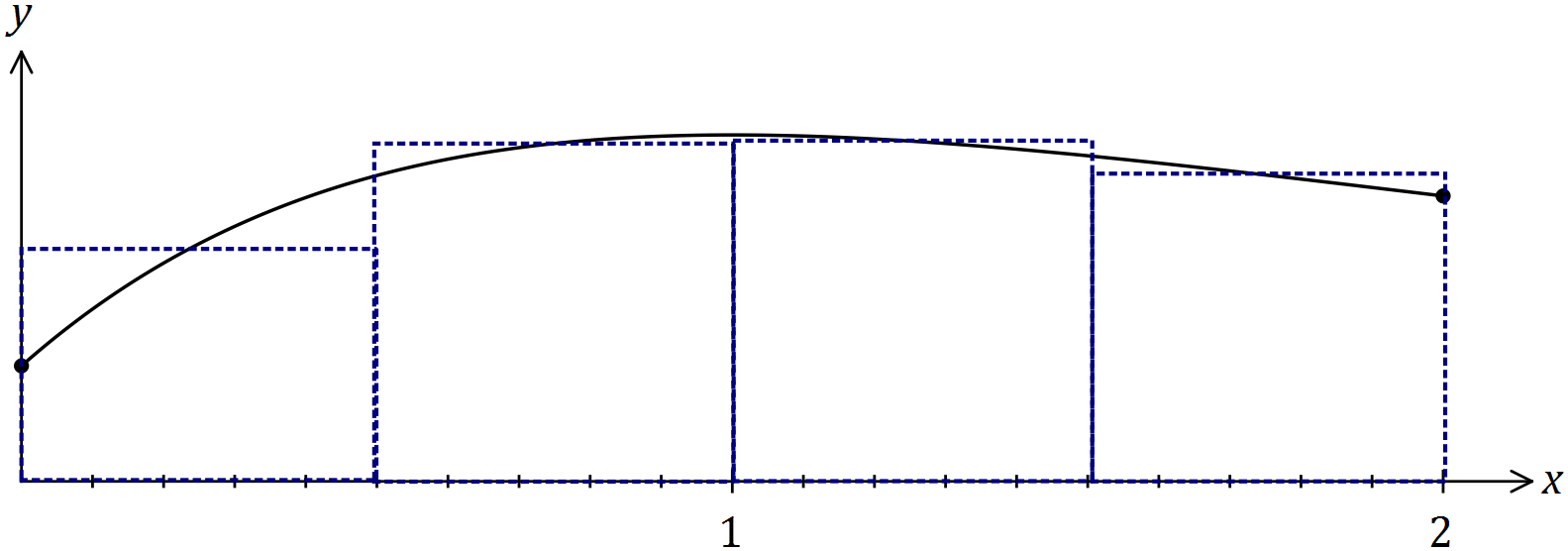
Section Two: Calculator-assumed 65% (97 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (8 marks)

The graph of is shown below for , where .



|  |
| --- |
| **Solution (b)(i)** |
| See graph |
| **Specific behaviours** |
| ✓ correct heights  ✓ correct centres |

(a) Show that has a stationary point at . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows derivative  ✓ shows has a factor of  ✓ indicates |

(b) An estimate for the area bounded by the curve, the -axis, the -axis and the line is required. A suitable estimate can be calculated from the sum of the areas of four centred rectangles with heights and , each with a width of units.

(i) Clearly show these four rectangles on the graph above. (2 marks)

(ii) Use the rectangles to estimate the area, giving your answer correct to 2 decimal places. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct heights  ✓ multiplies by width  ✓ correct area |

Question 10 (7 marks)

The capacity, mL, of glass bottles made in a factory can be modelled by a normal distribution with mean and standard deviation mL.

(a) If , determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states probability |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates numerator  ✓ states probability |

(iii) the value of , if . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states value |

(b) Given that ,

(i) determine the value of in terms of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation using correct -score  ✓ expression for , correct to 1 dp |

(ii) determine if . (1 mark)

|  |
| --- |
| **Solution** |
| mL |
| **Specific behaviours** |
| ✓ states value |

Question 11 (9 marks)

A fair die has one face numbered 1, two faces numbered 2 and three faces numbered 3.

(a) Determine the probability that the second even number occurs on the fourth throw of the dice. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses  ✓ uses binomial expansion for 1 even in 3 throws  ✓ calculates probability |

(b) The die is thrown twice and is the sum of the two scores.

(i) Complete the table below to show the probability distribution of . (2 marks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  ✓ |

(ii) Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours36** |
| ✓  ✓ probability |

(iii) Calculate . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sum of  ✓ simplifies |

Question 12 (8 marks)

From a random survey of 524 users of a free music streaming service, it was found that 386 would stop using it if they had to pay.

(a) Based on this survey, calculate the percentage of users who would stop using the service. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates percentage |

(b) Calculate the approximate margin of error for a 90% confidence interval estimate of the proportion of users who would stop using the service. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses correct -score  ✓ calculates standard error  ✓ calculates margin of error |

(c) Determine a 90% confidence interval for the proportion of users who would stop using the service. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes interval  ✓ rounds to 2, 3 or 4 decimal places |

(d) If 50 identical surveys were carried out and a 90% confidence interval for the proportion was calculated from each survey, determine the probability that exactly 48 of the intervals will contain the true value of the proportion. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states parameters of binomial distribution  ✓ calculates probability |

Question 13 (7 marks)

The lifetime, hours, of an electronic component is a continuous random variable with probability density function given by

(a) Determine the probability that a randomly chosen component has a lifetime of less than hours. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral  ✓ evaluates |

(b) An engineer buys 12 of the components. If they operate independently of each other, determine the probability that at least 11 of them will not last 450 hours. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates binomial distribution with parameters  ✓ calculates probability |

(c) A component has already been operating for exactly 440 hours. Determine the probability that it will fail within the next 36 hours. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  ✓  ✓ probability |

Question 14 (8 marks)

A researcher wants to estimate the proportion of Western Australian school-aged students who participate in organised sport during school holidays. The researcher plans to collect sample data by visiting schools and asking students.

(a) Discuss two different sources of bias that may occur when the researcher collects their sample data and suggest a procedure to avoid bias. (4 marks)

|  |
| --- |
| **Solution** |
| Undercoverage (*including volunteer or convenience sampling*) - the researcher should ensure that all students have an equal chance of being selected, rather than favouring gender, age, state, etc  Nonresponse - some students may choose not to answer the question  Etc, etc  To avoid bias use  Simple random sampling - number all students and select numbers at random  Systematic sampling - number all students and select every th student  Etc, etc |
| **Specific behaviours** |
| ✓ discusses one source of bias  ✓ discusses second source of bias  ✓ suggests a suitable type of sampling  ✓ explains sampling procedure |

(b) Determine, to the nearest 10, the sample size the researcher should use to ensure that the margin of error of a 90% confidence interval is no more than 6%. (3 marks)

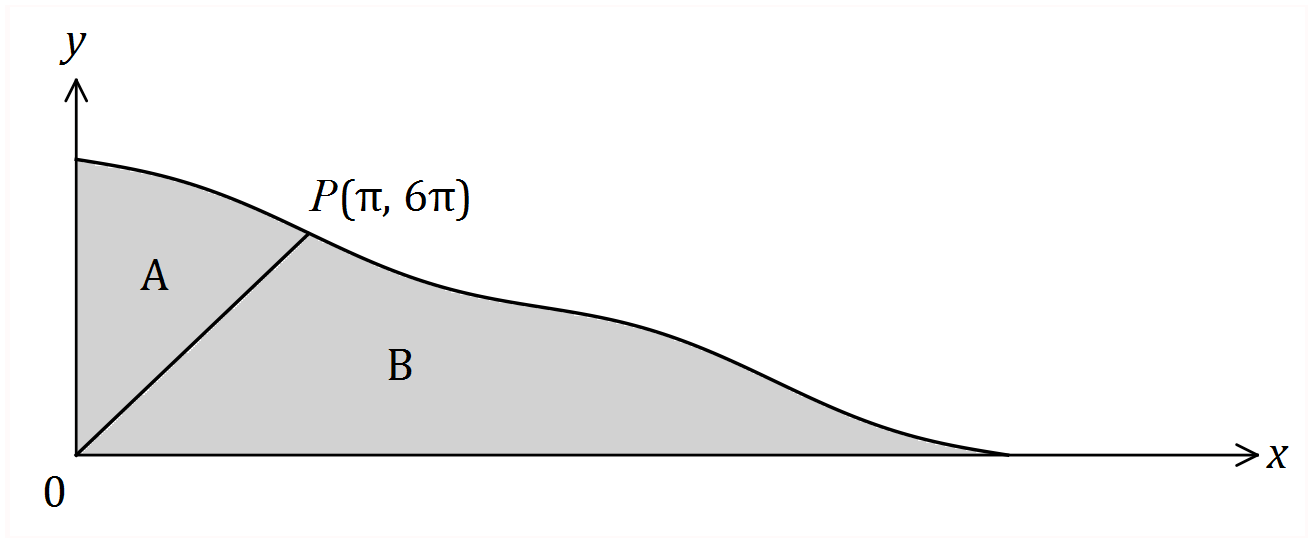
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ assumes  ✓ shows sample size equation  ✓ calculates |

(c) Comment on how your answer to (b) would change if the researcher had a reliable estimate that the population proportion was close to 20%. (1 mark)

|  |
| --- |
| **Solution** |
| Size of sample would decrease (*to close to 120*) |
| **Specific behaviours** |
| ✓ states decrease |

Question 15 (7 marks)

The curve is shown below passing through .



A straight line joins the origin to , dividing the shaded area into two regions, and .

(a) Show that when , . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes |

(b) Determine the value of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates integral  ✓ states exact value |

(c) Determine the ratio of the area of region to the area of region in the form .

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates area of triangle  ✓ evaluates area A  ✓ evaluates area B  ✓ states ratio in required form |

Question 16 (8 marks)

160 black and 840 white spherical beads, identical except for their colour, are placed in a container and thoroughly mixed.

In experiment , a bead is randomly selected, its colour noted and then replaced until a total of 20 beads have been selected.

(a) The random variable is the number of black beads selected in experiment .

Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates binomial RV, with parameters  ✓ states P |

(b) Experiment is repeated 10 times. Determine the probability that at least one black bead is selected in each of these experiments. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates P(at least one black) in one experiment  ✓ calculates probability |

In experiment , a bead is randomly selected, its colour noted and then replaced until a total of 65 beads have been selected.

Experiments and are repeated a large number of times, with the proportions of black beads in each experiment, and respectively, recorded.

(c) The distribution of which proportion, or , is most likely to approximate normality? Explain your answer and state the mean and standard deviation of the normal distribution for the proportion you have chosen. (4 marks)

|  |
| --- |
| **Solution** |
| most likely, as it is based on much larger sample size (*65 rather than 20*).  Parameters:  Mean:  Variance: , |
| **Specific behaviours** |
| ✓ chooses  ✓ explains is based on larger sample size  ✓ states mean  ✓ states standard deviation |

Question 17 (7 marks)

A polynomial function is such that

(a) Show that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ reverses limits and changes sign  ✓ factors and divides |

(b) Determine the value of . (5 marks)

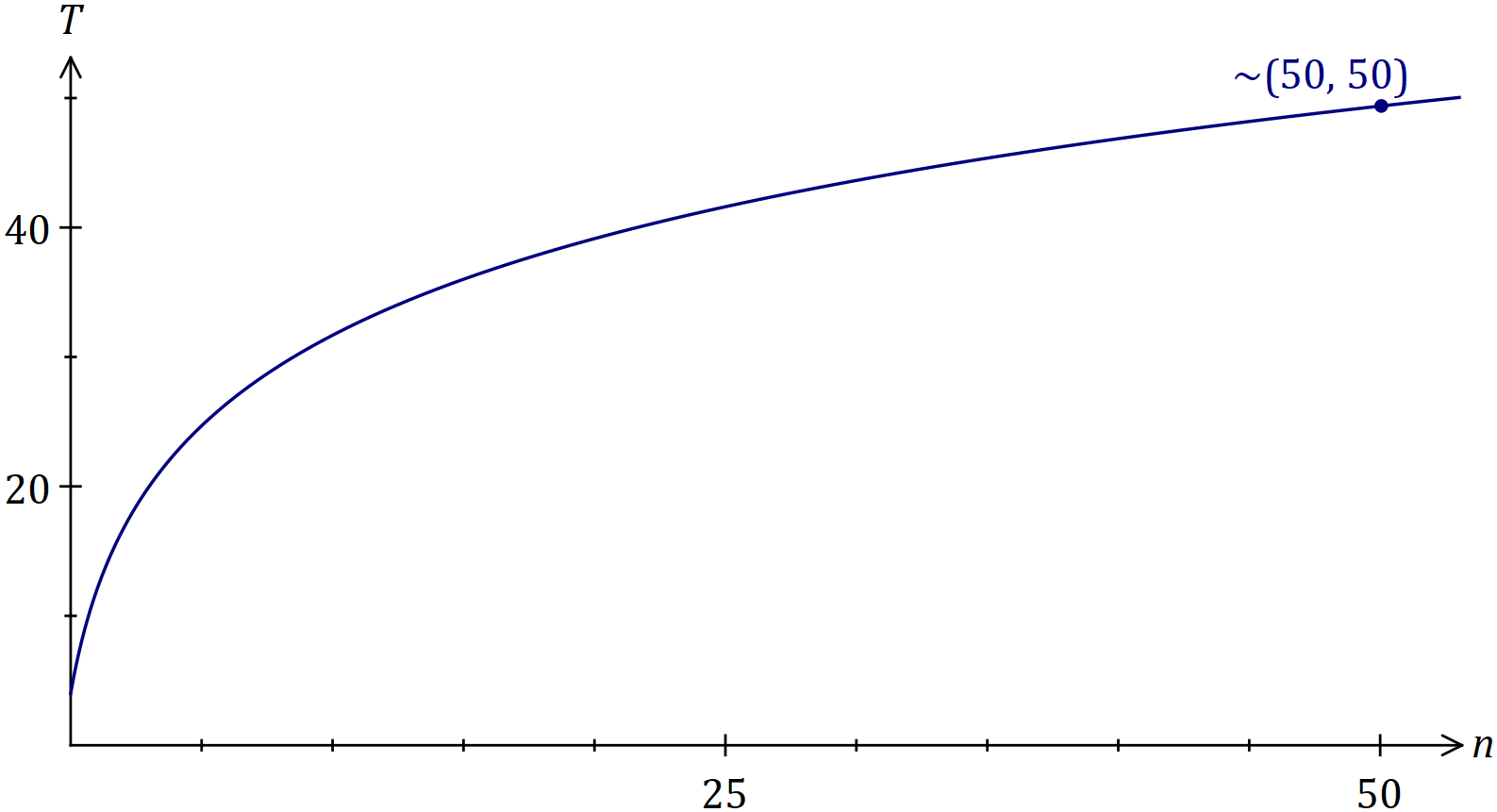
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses linearity to split  ✓ uses interval addition with  ✓ integrates  ✓ evaluates  ✓ correct sum |

Question 18 (8 marks)

Hick's law, shown below, models the average time, seconds, for a person to make a selection when presented with equally probable choices.

, where and are positive constants.

(a) Draw the graph of vs on the axes below when and . (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ adds scale to -axis  ✓ endpoints  ✓ log-shaped curve |

(b) When a pizzeria had 10 choices of pizza, the average time for patrons to make a choice was 40 seconds. After doubling the number of choices, the average time to make their choice increased by 25%.

Modelling the relationship with Hick's law, predict the average time to make a choice if patrons were offered a choice of 35 pizzas. (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes first equation  ✓ writes second equation  ✓ solves for variables  ✓ substitutes correctly  ✓ states time, rounded to nearest second |

Question 19 (8 marks)

The mass, g, of wasted metal when a cast is made is a random variable with probability density function given by

where is a positive constant.

(a) Determine in terms of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct integral  ✓ evaluates integral in terms of |

(b) The total mass of wasted metal from a random sample of 40 casts was 960 g. Estimate the value of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates sample mean  ✓ determines |

(c) If , determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates probability |

(ii) . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows value of  ✓ writes correct integral  ✓ evaluates variance |

Question 20 (6 marks)

A random sample of 510 rabbits from a nature reserve are captured, tagged and then set free. After a suitable interval, during which time it is assumed that the rabbit population does not change, another random sample of 300 rabbits is caught and 18 of these are observed to be tagged.

(a) Show that a point estimate for the size of the rabbit population is 8 500. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows use of direct proportion |

(b) Construct a 90% confidence interval for the proportion of rabbits in the population that are tagged. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates margin of error  ✓ states confidence interval |

(c) Deduce an approximate 90% confidence interval for the number of rabbits in the reserve.

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates lower value  ✓ calculates upper value  ✓ rounds sensibly (eg 00's) |

Question 21 (6 marks)

A popcorn container of capacity mL is made from paper and has the shape of an open inverted cone of radius and height .

Determine the least area of paper required to make the container. (6 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses in terms of and  ✓ expresses in terms of  ✓ expresses in terms of  ✓ differentiates  ✓ finds positive zero of derivative  ✓ substitutes to find minimum area |

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

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